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lentheorie, 4th edition, p. 38), we have for n odd,

$$2^{\phi(1)}-1\equiv 0(\mathrm{mod}\,1),\quad 2^{\phi(n)}-1\equiv 0(\mathrm{mod}\,n),\quad 2^{\phi(n')}-1\equiv 0(\mathrm{mod}\,n'),\ldots\ldots\ldots$$

where $\phi(n)$ is the number of integers less than n and prime to it. We have, then, since $\phi(1)=1$,

$$2^{\phi(1)}-1=1,\quad 2^{\phi(n)}-1\geqq n,\quad 2^{\phi(n')}-1\geqq n',\ldots\ldots\ldots$$

Now, unless $n=3$, there is at least one of the latter relations which is not an equality. For, let p be the largest prime factor of n, then $2^{p-1}-1\geqq p$.

Make $p=3$, then $2^{3-1}-1=3$. From this relation it is evident that $2^{p-1}-1>p$ for $p>3$. If $p=3$, then $n=3^k$. But

$$2^{\phi(3^2)}-1>3^2.$$

Hence, unless $k=1$ or $n=3$, there is at least one inequality in the original scheme, and we get by transposition and multiplication,

$$2^{\phi(1)+\phi(n)+\phi(n')\ldots\ldots}>2(n+1)(n'+1)(n''+1)\ldots\ldots\ldots$$

or, since $\phi(1)+\phi(n)+\phi(n')\ldots\ldots=n$ (Dirichlet, l. c., p. 26),

$$2^n>2(n+1)(n'+1)(n''+1)\ldots\ldots\ldots$$

for $n>3$, which is the theorem.

118. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find the two least *positive** integral numbers such that their sum shall be a square and the sum of their squares a biquadrate.

Professor Walker finds that the two numbers,

$$x=4,565,468,027,761,\quad y=1,061,625,293,520$$

have the properties that their sum is a square and the sum of their sqnares is a biquadrate. Indeed, it may be verified that $(x+y)^{\frac{1}{2}}=2372159$; $(x^2+y^2)^{\frac{1}{4}}=2165017$. In the letter accompanying the solution Professor Walker asks "are these the least numbers?" We invite other solutions bearing upon this important point of the problem.

119. Proposed by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, The University of Chicago.

If p be any prime number and n any positive integer, the congruence $x^{p^n}\equiv x(\mathrm{mod}\,p^n)$ has p and only p solutions $\mathrm{mod}\,p^n$. Hence the congruence defines the Galois field of order p^n if and only if $n=1$.

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*As originally printed the word "positive" was omitted.

Solution by the PROPOSER.

The only solution x divisible by p is seen to be $\equiv 0 (\mathrm{mod} p^n)$. Let next x be prime to p. Then

$$x^{p^n-1} \equiv 1(\mathrm{mod} p^n), \quad x^{p^{n-2}(p-1)} \equiv 1(\mathrm{mod} p^n),$$

the second from Fermat's theorem generalized, since $\phi(p^n)=p^{n-1}(p-1)$. Hence must $x^{p-1} \equiv 1(\mathrm{mod} p^n)$. To prove that the latter has precisely $p-1$ distinct solutions $\mathrm{mod} p^n$, we proceed by induction, the result being true for $n=1$ by Fermat's theorem. Let therefore $x_1, \ldots\ldots, x_{p-1}$ be the distinct roots of $x^{p-1} \equiv 1(\mathrm{mod} p^a)$, whence $x_i^p = x_i + r_i p^a$, each r_i being a fixed integer. To determine the roots of $x^{p-1} \equiv 1(\mathrm{mod} p^{a+1})$, we seek the integral values of m_i incongruent $\mathrm{mod} p$ such that $x = x_i + m_i p^a$ satifies $x^p \equiv x(\mathrm{mod} p^{a+1})$. But $(x_i + m_i p^a)^p = x_i^p +$ multiple of p^{a+1}, by the Binomial Theorem. The condition is therefore

$$x_i + r_i p^a \equiv x_i + m_i p^a \ (\mathrm{mod} p^{a+1}),$$

whence $m_i \equiv r_i (\mathrm{mod} p)$ and m_i is uniquely determined modulo p.

AVERAGE AND PROBABILITY.

148. Proposed by M. C. RORTY, Boston, Mass.

Assuming n points to fall at random upon a circle of circumference a, what is the probability of m or more points falling within a length b upon this circumference?

NOTE. This problem has practical application in determining the probability of accidental rushes of telephone calls as distinct from those rushes which are due to commercial causes. The solution for m or more points falling within a specified length b is known. The problem presented above differs from this in that a solution is required for *any* length b.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $m+r$, $(r=0, 1, 2, 3, \ldots\ldots n-m)$ be the number of points to fall on b. $m+r$ points can be selected from n points in $n! \div (m+r)!(n-m-r)!$ ways.

$$\therefore \text{Chance} = \frac{2n!}{(m+r)!(n-m-r)!} \cdot \frac{(b)^{m+r}(a-b)^{n-m-r}}{a^n}.$$

149. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Three points are taken at random on the convex surface of a right cone. Find the probability that the section of the cone made by the plane passing through them is a complete ellipse.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be a section of the cone, E one of the random points in the surface. $AD=c$, $GD=h$, $BD=R$, $EG=r$. Through E pass the plane EC. Then the area of the surface, of which EBC is a projection

$$= \frac{\pi \sqrt{[h^2 + (R-r)^2]}}{R-r} [R^2 - (R+r)\sqrt{(Rr)}].$$